Algebra and Calculus: Worksheet 7

Graphing rational functions using transformations:

In general, we can represent rational functions that are ratios of first degree polynomials as transformations of the function $f(x) = \frac{1}{x}$:

$$\begin{aligned}\frac{ax+b}{cx+d} &= \frac{a}{c} + \frac{b - \frac{ad}{c}}{c\left(x+\frac{d}{c}\right)}\\ &= \frac{a}{c} + \left(b - \frac{ad}{c}\right) f\left(c\left[x + \frac{d}{c}\right]\right)\end{aligned}$$

assuming $c \neq 0$. This gives the horizontal and vertical asymptotes immediately:

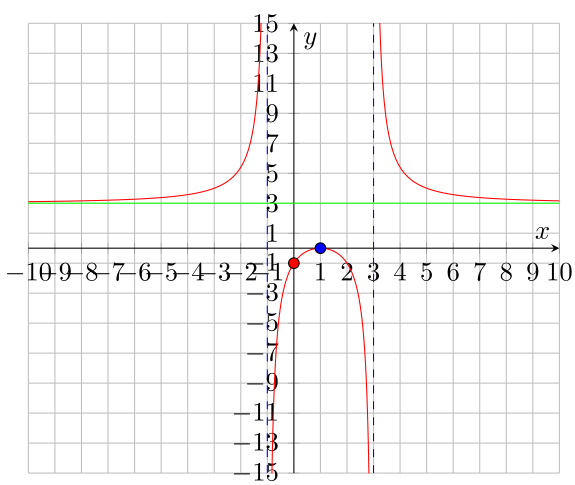
- horizontal: $y = \frac{a}{c}$
- vertical: $x = -\frac{d}{c}$

The formula is simply the result of dividing $cx+d$ into $ax+b$, and you can always use that strategy if you forget the formula.

1. $t(x) = \frac{3x-3}{x+2}$.

2. Determine the x and y intercepts and the vertical and horizontal asymptotes.

   The graph is of a rational function equal to the ratio of two quadratic polynomials. Now use all the given information to find the function.



3. Find the slant asymptotes and vertical asymptotes of $r(x) = \frac{x^2}{x-2}$.

4. Find all relevant quantities (intercepts and asymptotes) and plot the graph:

$$r(x) = \frac{x^2 + 4x - 5}{x^2 + x - 2}$$

5. **True or false**: If there is a vertical asymptote at $x = c$, then the domain must not include the point $x = c$.

6. **True or false**: If there is a horizontal asymptote at $y = d$, then the range must not include the point $y = d$.

7. *Q87, Section 3.6*: Suppose that the rabbit population on Mr. Jenkins' farm follows the formula

$$p(t) = \frac{3000t}{t+1}$$

   where $t \geq 0$ is in months.

   (a) Draw a graph of the rabbit population

   (b) What eventually happens to the population?

8. (*Q94 Section 3.6*): Explain how you can tell (without graphing it) that the function

$$r(x) = \frac{x^6 + 10}{x^4 + 8x^2 + 15}$$

   has no x-intercept and no horizontal, vertical, or slant asymptote?

9. it True or False: The rational function $\frac{x+2}{x^2-4}$ has vertical asymptotes at $x = \pm 2$.