Algebra and Calculus: Quiz 4 (Solutions)

Name/NetID: ____________________________

Complete all problems.

1. For **multiple choice** problems, circle the letter corresponding to the correct answer.
2. For **free response** problems, **show all work** and put a $\boxed{\text{box}}$ around your final answer.

Good luck!

Answers: D, B, A, C, [see graph]

1. All of the following describe one-to-one functions EXCEPT
   (a) If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.
   (b) If $f(x_1) = f(x_2)$, then $x_1 = x_2$.
   (c) No horizontal line intersects its graph more than once.
   (d) No vertical line intersects its graph more than once.

   Solution: The answer is $\boxed{\text{(d)}}$. Note that (d) is the vertical line test, which is a test for a **function**, and not just a one-to-one function. Answer (c) is the horizontal line test which is indeed a test for one-to-one functions.

   Answer (a) says that two distinct inputs must produce two distinct outputs, and (b) says that two identical outputs could only have been produced from two identical inputs. These statements are actually completely equivalent, and are thus both valid ways of testing whether a function is one-to-one.

2. If $f(x) = \dfrac{2x-1}{x+1}$, then $f^{-1}(1) =$
   (a) 1
   (b) 2
   (c) -1
   (d) -2

   Solution: Let's rewrite the expression we're asked for:

$$x = f^{-1}(1)$$
$$\implies f(x) = 1$$

Thus, we are looking for the value of x such that $f(x) = 1$. Alternatively one could solve for the inverse function and evaluate it at $x = 1$. But given that this is a multiple choice question, it makes more sense to plug in values until we get the answer we're looking for. Plugging in $x = 2$,

$$\begin{aligned} f(2) &= \frac{2(2)-1}{2+1} \\ &= \frac{4-1}{2+1} \\ &= \frac{3}{3} = 1 \end{aligned}$$

so the answer is $\boxed{\text{(b)}}$.

3. The standard form of $-x^2 + 4x - 3$ is
   (a) $-(x-2)^2 + 1$
   (b) $-(x-2)^2 - 1$
   (c) $-(x+2)^2 + 1$
   (d) $-(x+2)^2 - 1$

   Solution: Let's solve this the usual way:

$$\begin{aligned} -x^2 + 4x - 3 &= -(x^2 - 4x) - 3 \\ &= -(x^2 - 4x + 4 - 4) - 3 \\ &= -(x^2 - 4x + 4) + 4 - 3 \\ &= -(x-2)^2 + 1 \end{aligned}$$

   So the answer is $\boxed{\text{(a)}}$.

Note: There is a formula to write a quadratic polynomial ($f(x) = ax^2 + bx + c$) in standard form:

$f(x) = a\left(x + \frac{b}{2a}\right)^2 + f\left(-\frac{b}{2a}\right)$

4. The vertex of $-x^2 + 4x - 3$ is
   (a) $(-2, -1)$
   (b) $(-2, 1)$
   (c) $(2, 1)$
   (d) $(2, -1)$

   Solution: The easiest way to do this is to go back to the previous question (given that you've solved it correctly!) and identify the vertices. In vertex/standard form:

$$-(x-2)^2 + 1 = a(x-r)^2 + s$$

Where a is the coefficient on the quadratic, c is the x-coordinate of the vertex and b is the y-coordinate of the vertex. We have $r = 2$ and $s = 1$, so our vertex is $(r, s) = (2, 1)$.

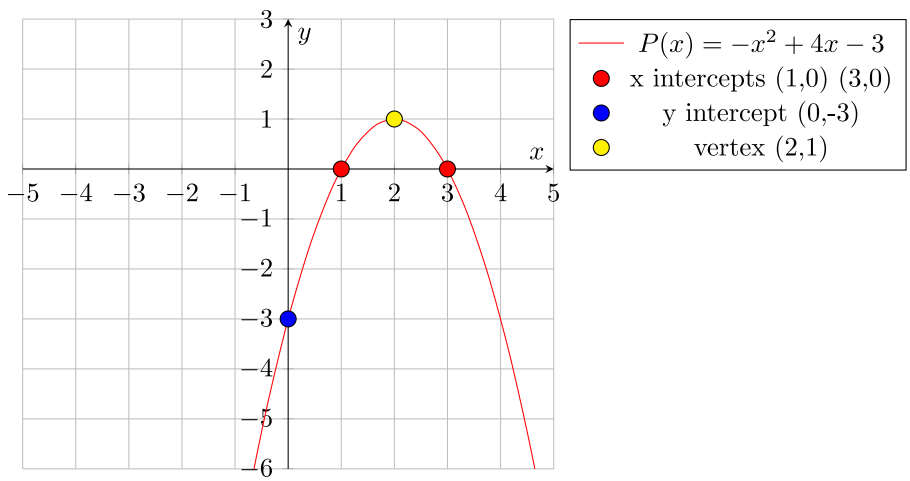
So the answer is $\boxed{\text{(c)}}$.

5. Graph $-x^2 + 4x - 3$, labeling the vertex, x- and y-intercepts. Use the graph to determine the domain and range of the function.

Solution: The previous question gave us the location of the vertex, which is at $(2, 1)$. The x-intercepts and y intercept may be determined from the form of the polynomial. Let $f(x) = -x^2 + 4x - 3$. Then

$$\text{(y-intercept) } f(0) = -3$$

$$\begin{aligned} \text{(x-intercept) } f(x) &= 0 \\ -x^2 + 4x - 3 &= 0 \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \\ x &= \{1, 3\} \end{aligned}$$



The graph of the function tells us the domain and range. Due to the fact that any value of x on the real line is valid, the domain is given by $x \in \mathbb{R}$. As for the range, we see that we have a maximum at the vertex, which means that the upper bound on the range is $y = 1$. There is no lower bound, so we may have $y \to -\infty$. Thus, the domain and range are

$$\begin{aligned} -\infty < x < \infty \\ -\infty < y \leq 1 \end{aligned}$$