Algebra and Calculus: Homework 6 Solutions

Section 2.8: 16,18,44,58

- *Q16*: Determine whether the function is one-to-one: $g(x) = |x|$.

  If a function is one-to-one, then we should not be able to find two values of x, a and b, such that $a \neq b$ and $g(a) = g(b)$. But take a to be any positive real number other than 0 and let $b = -a$. Then

  $$\begin{aligned} g(a) &= |a| = a \\ g(b) &= g(-a) = |-a| = a \end{aligned}$$

  So $g(a) = g(b)$ and thus this function is not one-to-one.

- *Q18*: Determine whether the function is one-to-one: $h(x) = x^3 + 8$.

  Let us find the inverse function $h^{-1}(x)$:

  $$\begin{aligned} y &= h(x) = x^3 + 8 \\ x^3 &= y - 8 \\ x &= (y-8)^{\frac{1}{3}} \\ h^{-1}(x) &= (x-8)^{\frac{1}{3}} \end{aligned}$$

  The domain of the original function was all real numbers, and so was the range. Thus, for the inverse function the domain is also all real numbers (the cube root may take negative arguments), and from the form of the function we can see that the range of $h^{-1}(x)$ is also all real numbers.

  Thus, this is a one-to-one function.

- *Q44*: Use the inverse function property to show that f and g are inverses of each other:

  $$\begin{aligned} f(x) &= x^3 + 1 \\ g(x) &= (x-1)^{\frac{1}{3}} \end{aligned}$$

  If the two functions are inverses, then $f(g(x)) = g(f(x)) = x$. Let's compute both:

  $$\begin{aligned} f(g(x)) &= \left[(x-1)^{\frac{1}{3}}\right]^3 + 1 \\ &= x - 1 + 1 = x \\ g(f(x)) &= (x^3 + 1 - 1)^{\frac{1}{3}} \\ &= (x^3)^{\frac{1}{3}} = x \end{aligned}$$

  So in fact f and g are inverses.

- *Q58*: Find the inverse function of f: $f(x) = \frac{4x-2}{3x+1}$

  Let $y = f^{-1}(x)$. Then

$$\begin{aligned} x &= \frac{4y-2}{3y+1} \\ x(3y+1) &= 4y-2 \\ (3x)y + x &= 4y-2 \\ (3x-4)y &= -(x+2) \\ y &= \frac{x+2}{4-3x} \end{aligned}$$

Section 3.1: 18, 22

- *Q18*: A quadratic function f is given: $f(x) = -x^2 - 4x + 4$

  1. Express f in standard form.

     We proceed:

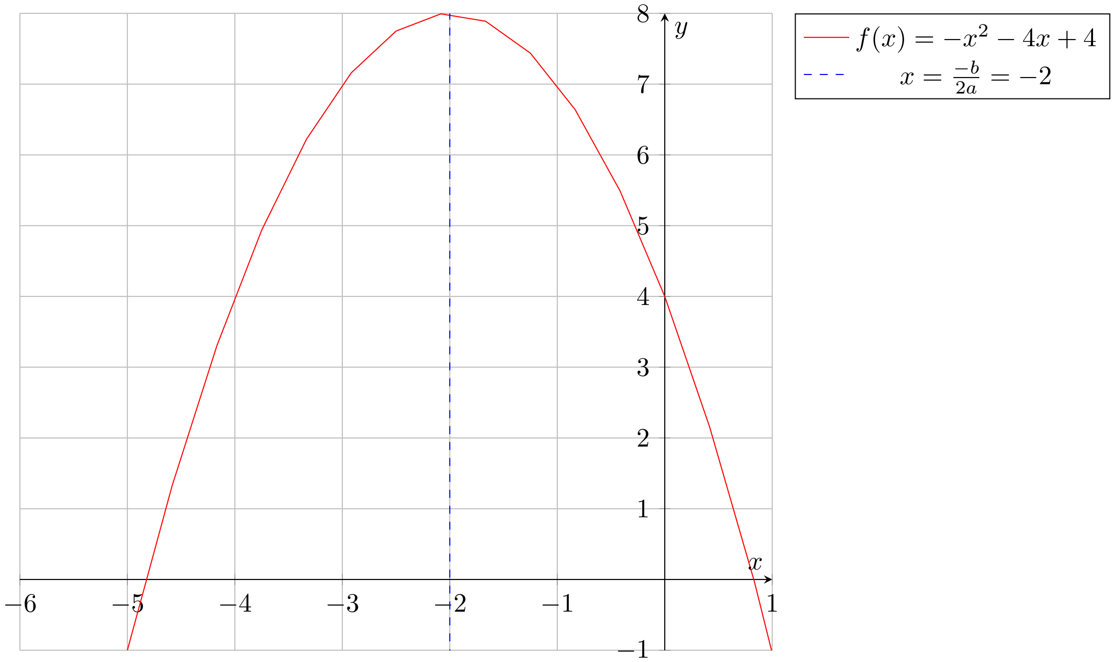
$$\begin{aligned} f(x) &= -(x^2+4x)+4 \\ &= -(x^2+4x+4-4)+4 \\ &= -(x^2+4x+4)+4+4 \\ &= -(x+2)^2+8 \end{aligned}$$

  2. Find the vertex and x- and y-intercepts of f.

     The y-intercept is given by $f(0) = -0^2 - 4(0) + 4 = 4$. The x-intercepts are given by the roots:

$$\begin{aligned} f(x) &= -(x^2+4x-4) = 0 \\ x^2+4x-4 &= 0 \\ x &= \frac{-4 \pm \sqrt{4^2-4(1)(-4)}}{2} \\ &= \frac{-4 \pm \sqrt{16+16}}{2} \\ &= \frac{-4 \pm \sqrt{32}}{2} \\ &= \frac{-4 \pm 4\sqrt{2}}{2} \\ &= -2 \pm 2\sqrt{2} \end{aligned}$$

  3. Sketch a graph of f.



4. The range is not bounded below, but is bounded above by the maximum value, which we can see is $f(-2) = 8$. The domain is unrestricted, so all $x \in \mathbb{R}$. Thus

$$-\infty < x < \infty$$
$$-\infty < f(x) \leq 8$$

- *Q22*: A quadratic function f is given: $f(x) = 2x^2 + 12x + 10$

  1. Express f in standard form.

     We proceed:

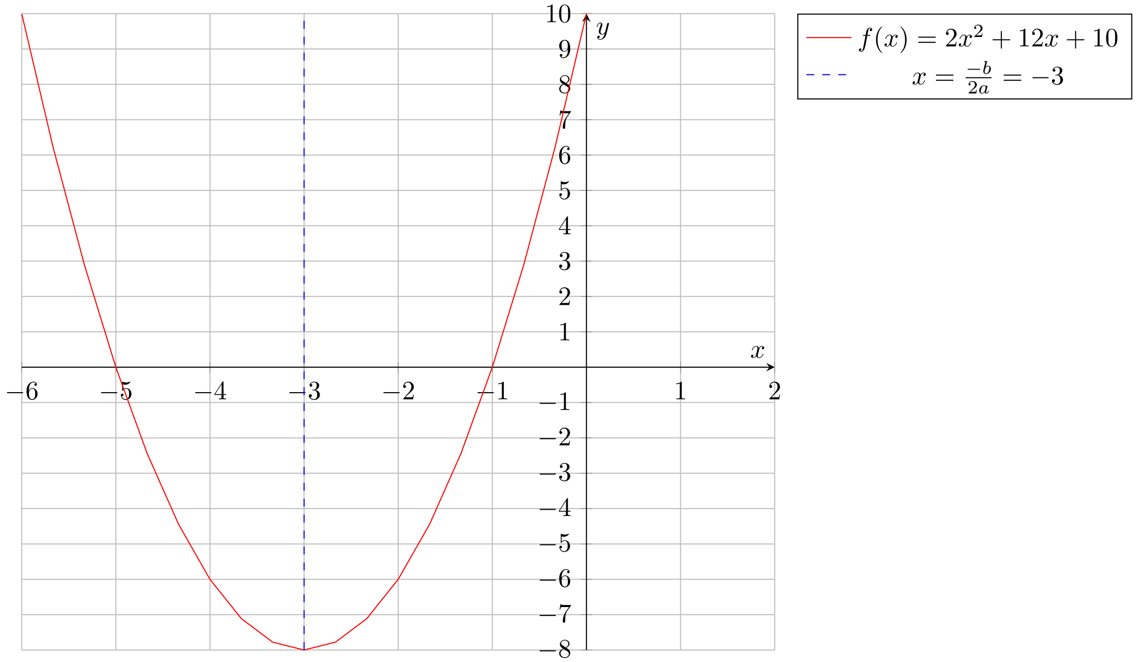
$$\begin{aligned} f(x) &= 2(x^2 + 6x) + 10 \\ &= 2(x^2 + 6x + 9 - 9) + 10 \\ &= 2(x^2 + 6x + 9) - 18 + 10 \\ &= 2(x + 3)^2 - 8 \end{aligned}$$

  2. Find the vertex and x- and y-intercepts of f.

     The y-intercept is given by $f(0) = 2(0)^2 + 12(0) + 10 = 10$. The x-intercepts are given by the roots:

$$\begin{aligned} f(x) = 2x^2 + 12x + 10 &= 0 \\ x^2 + 6x + 5 &= 0 \\ (x + 5)(x + 1) &= 0 \\ x &= \{-5, -1\} \end{aligned}$$

  3. Sketch a graph of f.



4. The range is not bounded above, but is bounded above by the minimum value, which we can see is $f(-3) = -8$. The domain is unrestricted, so all $x \in \mathbb{R}$. Thus

$$-\infty < x < \infty$$
$$-8 \leq f(x) < \infty$$