Algebra and Calculus: Homework 2 Solutions

Section 2.1: 48,70

- *Q48*: Find $f(a), f(a+h)$, and the difference quotient $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$.

$$
\begin{aligned}
f(x) &= \frac{2x}{x-1} \\
f(a) &= \frac{2a}{a-1} \\
f(a+h) &= \frac{2(a+h)}{a+h-1} \\
\frac{f(a+h)-f(a)}{h} &= \frac{1}{h}\left(\frac{2(a+h)}{a+h-1} - \frac{2a}{a-1}\right) \\
&= \frac{1}{h}\left(\frac{2(a+h)(a-1)}{(a+h-1)(a-1)} - \frac{2a(a+h-1)}{(a-1)(a+h-1)}\right) \\
&= \frac{1}{h}\left(\frac{2(a+h)(a-1) - 2a(a+h-1)}{(a-1)(a+h-1)}\right) \\
&= \frac{1}{h}\left(\frac{\cancel{-2a(a+h)} + 2a + \cancel{2a(a+h)} - 2(a+h)}{(a-1)(a+h-1)}\right) \\
&= \frac{1}{h}\left(\frac{\cancel{2a - 2a} - 2h}{(a-1)(a+h-1)}\right) \\
&= \frac{-2\cancel{h}}{\cancel{h}\left[(a-1)(a+h-1)\right]} \\
&= \frac{-2}{(a-1)(a+h-1)}
\end{aligned}
$$

- *Q70*: Find the domain: $f(x) = \frac{x^2}{6-x}$.

  Our domain is constrained by the fact that (1) the denominator cannot be equal to 0, and (2) the argument under a square root must be greater than or equal to 0. Putting these two constraints together, the argument under the square root must be greater than 0.

$$6 - x > 0 \implies x < 6$$

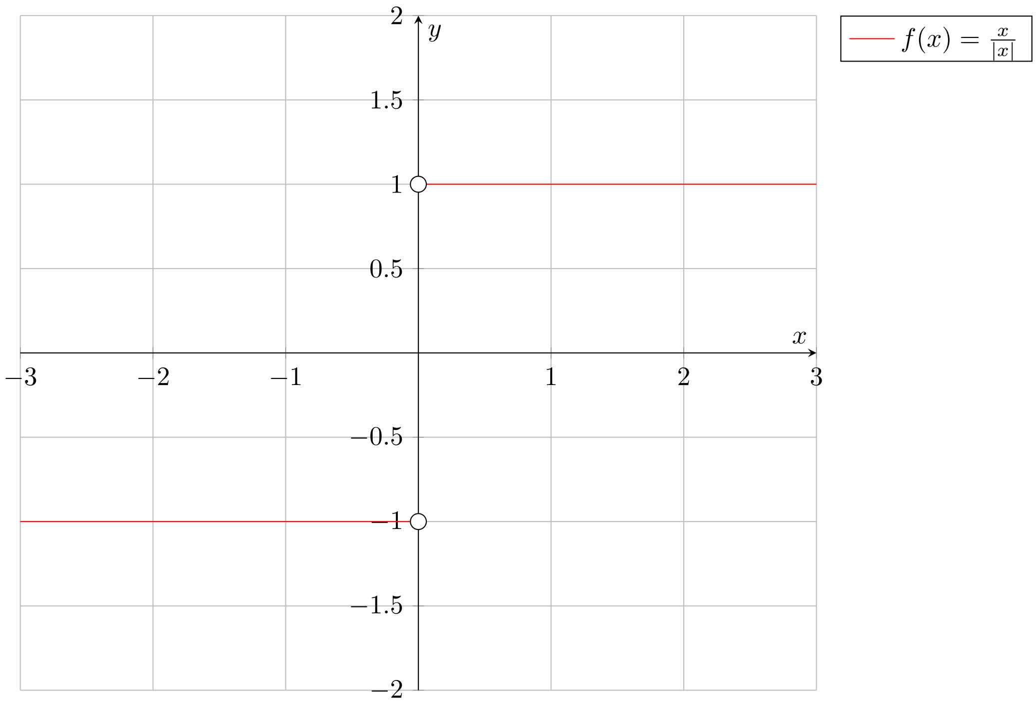
Section 2.2: 28,44

- *Q28* Sketch a graph by first making a table of values: $f(x) = \frac{x}{|x|}$.

We'll consider the interval $[-3, 3]$ (excluding 0, which is undefined and must be excluded from the domain):

x	f(x)
-3	-1
-2	-1
-1	-1
1	1
2	1
3	1

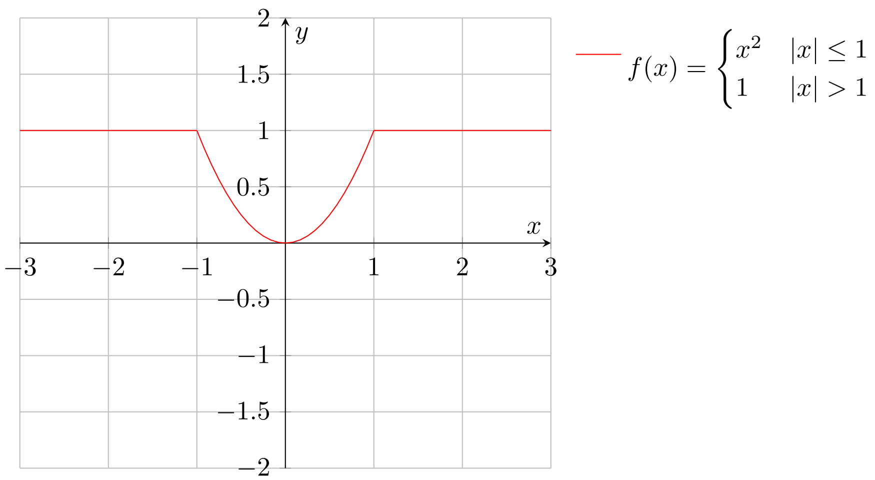
A graph:



- *Q44* Sketch a graph of the piecewise function:

$$f(x) = \begin{cases} x^2 & |x| \leq 1 \\ 1 & |x| > 1 \end{cases}$$

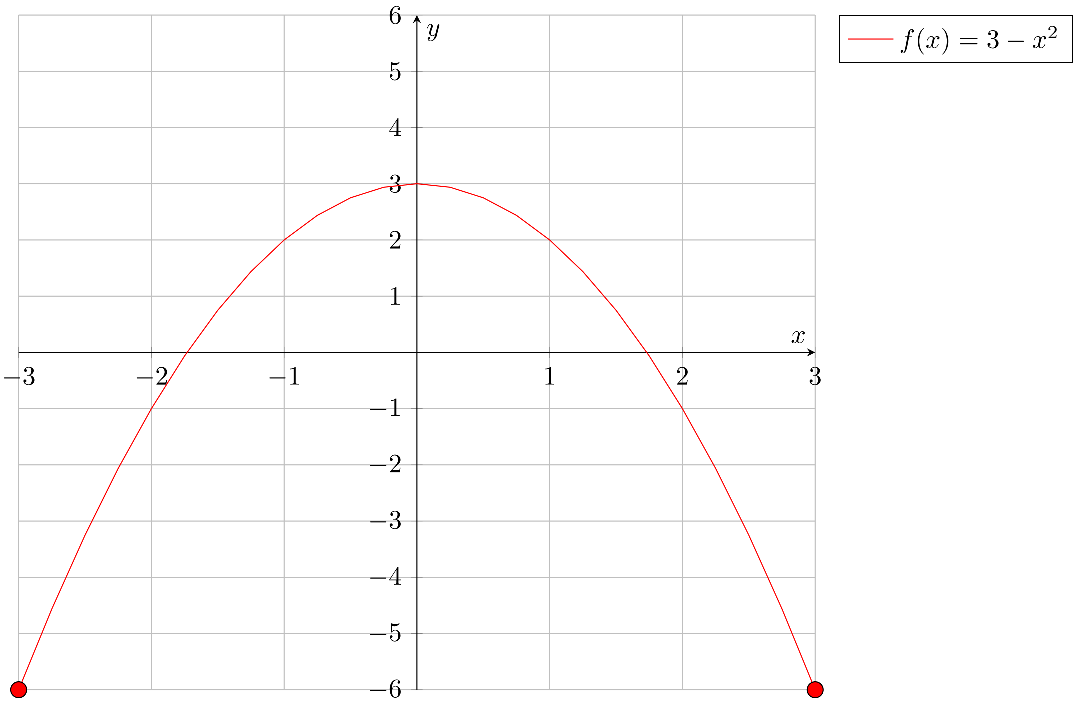
The graph:



Section 2.3: 16,28

- *Q16* Sketch a graph of f, and use the graph to find the domain and range of f: $f(x) = 3 - x^2$, $-3 \leq x \leq 3$.

  A graph:

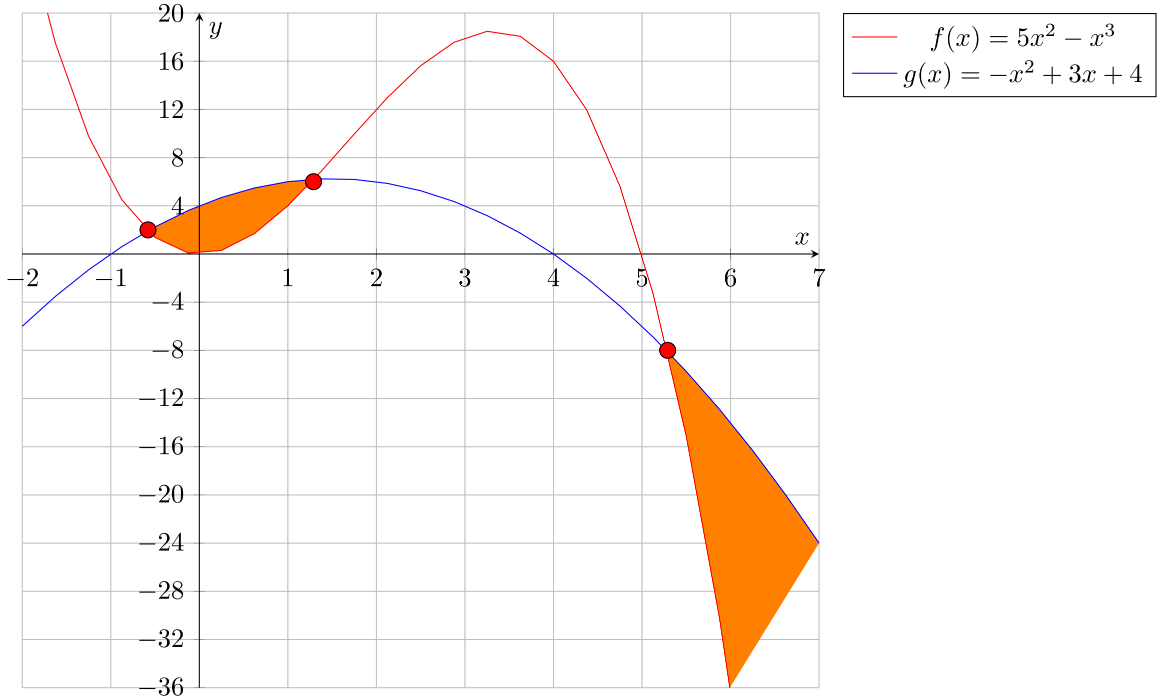


The domain is clearly given by $-3 \leq x \leq 3$, as this is specified. What is the range? Note that because $x^2 \geq 0$, with equality only when $x = 0$, $3 - x^2 \geq 0$. Thus the upper bound of the range is $y = 3$. What is the lower bound? This occurs at the edges of the domain, when $x = \pm 3$. The value is $3 - (3)^2 = -6$. Thus:

$$-6 \leq y \leq 3$$

- *Q28*: Solve equation and inequality graphically: $5x^2 - x^3 = -x^2 + 3x + 4$ and $5x^2 - x^3 \leq -x^2 + 3x + 4$.

  Let's graph the two functions. We'll let $f(x) = 5x^2 - x^3$ and $g(x) = -x^2 + 3x + 4$:



  So we see that the two graphs intersect at three points, given by the following x-values:

$$x \approx -0.58$$
$$x \approx 1.29$$
$$x \approx 5.29$$

  As for the inequality, we are looking for the regions where $f(x) \leq g(x)$. From looking at the graph, we see that this happens when

$$-0.58 \leq x \leq 1.29$$
$$x \geq 5.29$$